HYBRID MODELING OF CHAOTIC SYSTEMS WITH UNCERTAINTY QUANTIFICATION

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Abstract

Traditionally, modeling (parametric) and model-free (non-parametric) techniques are used for prediction, however it is uncommon for the two to be incorporated together. We compare the forecast accuracy of a Bayesian parametric methodology (DRAM) and a non-parametric approach (SSR) against each other as well as against a hybrid composed of the the two on a chaotic coupled dynamical system. We apply our hybrid approach on an age-structured population system using data from cannibalistic flour beetles, in which it has been observed that the adults preying on the eggs and pupae results in chaotic population dynamics.

Non-parametric: State Space Reconstruction

- Takens' Theorem gives a one-to-one mapping between the attractor manifold M of the full system, and its reconstructed "shadow" manifolds. [1]
- M is one-to-one with its shadow manifolds. Hence, the shadow manifolds can be used for forecasting the future state.
- We use a nearest neighbor approach to estimate future system states directly from data without using a mechanistic model.

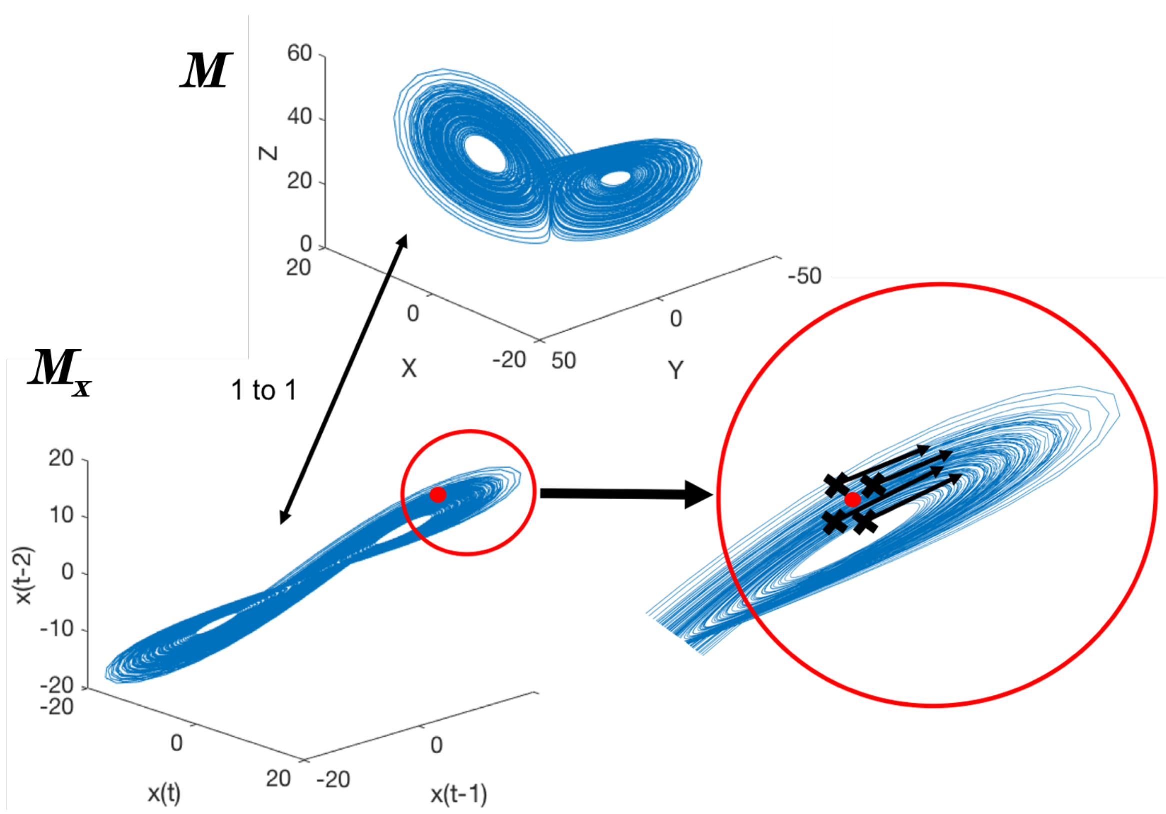


Figure: State Space Reconstruction on the Lorenz System.

- Prediction for an unmodeled state $x_m(t)$ is made by computing a weighted average of the target variable over its nearest neighbors:

$$\hat{x}_m(t+h) = \frac{\sum_{i=1}^{r} w_i(t) x_{N(t,i)+h}}{\sum_{i=1}^{r} w_i(t)}.$$

- The values $x_{N(t,i)+h}$ are used as a sample space for predictions, each with probability $p_i(t) = \frac{w_i(t)}{\sum_{i=1}^{r} w_i(t)}$, where $\mathbf{var}(x_m(t+h)) = \mathbf{E}((x_{N(t,i)+h} - \hat{x}_{t+h})^2)$.

Parametric: Bayesian Estimation with DRAM

- The Metropolis-Hastings type Markov chain Monte Carlo (MCMC) algorithms employ a Bayesian methodology and use delayed rejection and adaptive metropolis (DRAM) to obtain posterior distributions for the modeled variable parameters. [2]

Cannibalistic Beetle Population

Age-structured Population Model

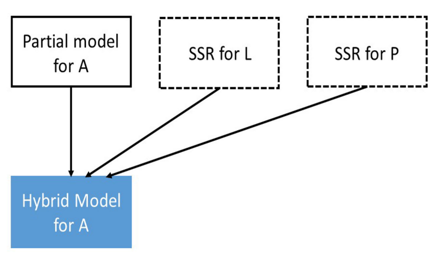
$$L(t) = bA(t-1)\exp(-C_{el}L(t-1) - C_{ea}A(t-1))$$
$$P(t) = L(t-1)(1-\mu_1)$$
$$A(t) = P(t-1)\exp(-C_{pa}A(t-1)) + A(t-1)(1-\mu_a)$$

Figure: Life stages of the flour beetle (from left to right): Egg, Larvae, Pupae, and Adults. [4]

Experimental Data

- Experimentally altered adult mortality, c_{pa}, to 7 different values, resulting in chaotic data sets. [3]
- 3 replicates per c_{pa}, total of 21 data sets.
- 41 time points per data set, sampled every 2 weeks.
- Total counts of Larvae, Pupae, and Adults.

Hybrid Methodology



Method Outline

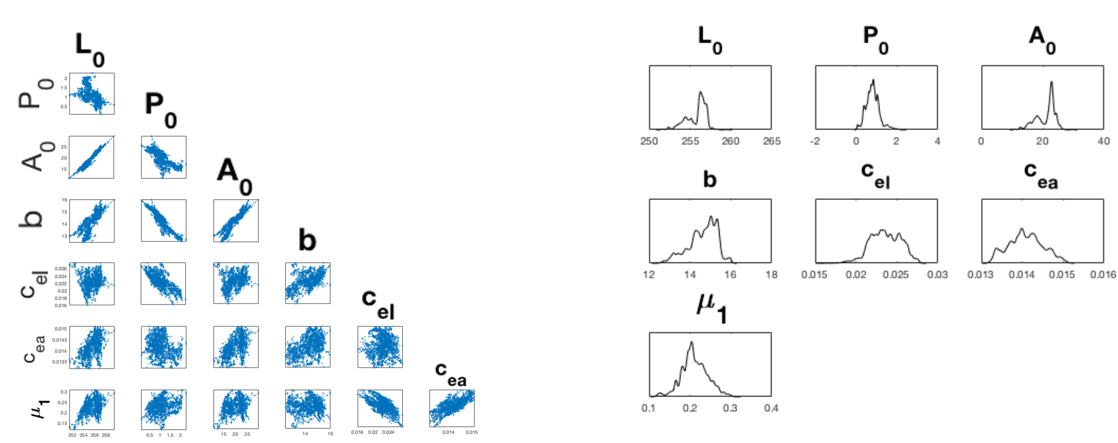
- The hybrid prediction for A uses a partial model for A and data or SSR for L and P, where

$$A(t) = P(t-1)\exp(-C_{pa}A(t-1)) + A(t-1)(1-\mu_a).$$

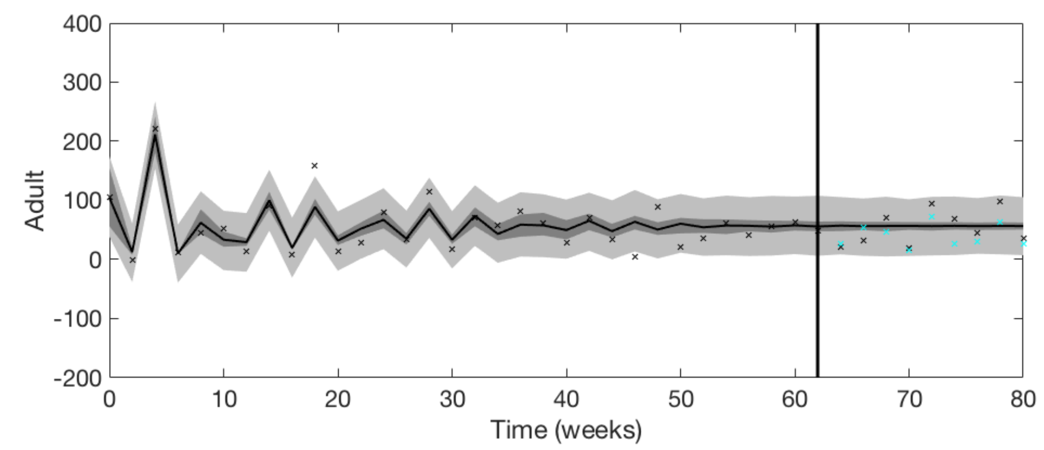
UQ for Hybrid Prediction

- Jointly sample from the posterior distributions of the parameters for the modeled variables and the SSR sample space.

Full Model Results



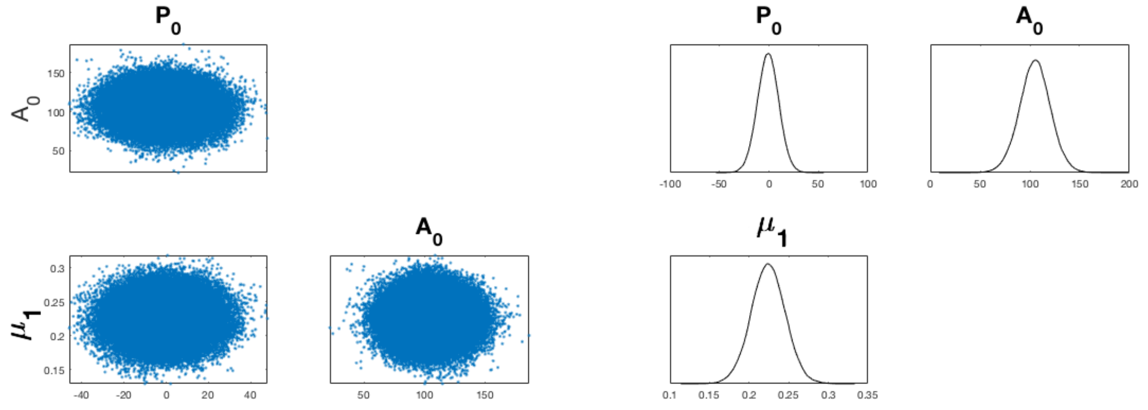
(a) Correlation Plots. (b) Posterior Distributions.



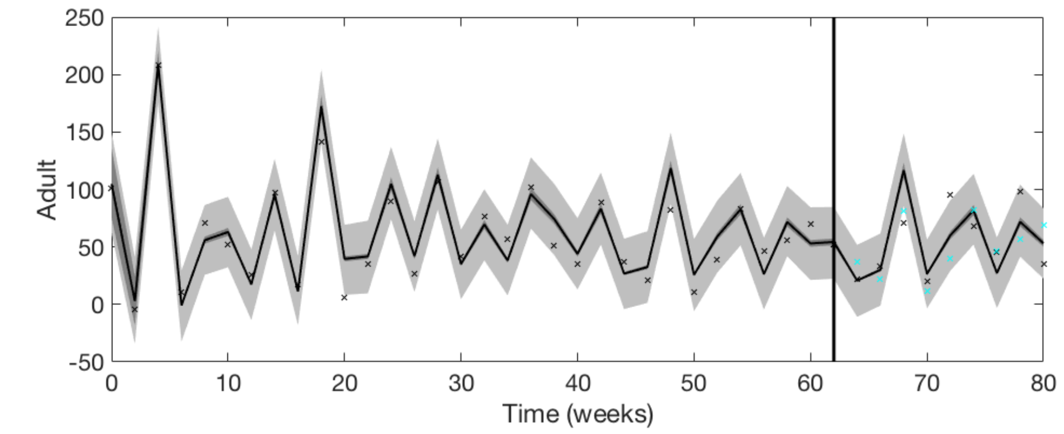
(c) Full Model Uncertainty Quantification for data set #1 and $C_{pa} = 0$.

- Time is measured in weeks, data points are recorded every 2 weeks.
- The vertical line indicates the end of the experimental data used for the training set (62 weeks) and the beginning of the testing set (18 weeks).

Hybrid Approach Results



(a) Correlation Plots. (b) Posterior Distributions.



(c) Hybrid Model, Model(A)/SSR(LP), Uncertainty Quantification for data set #1 and $C_{pa} = 0$.

Forecast Comparison

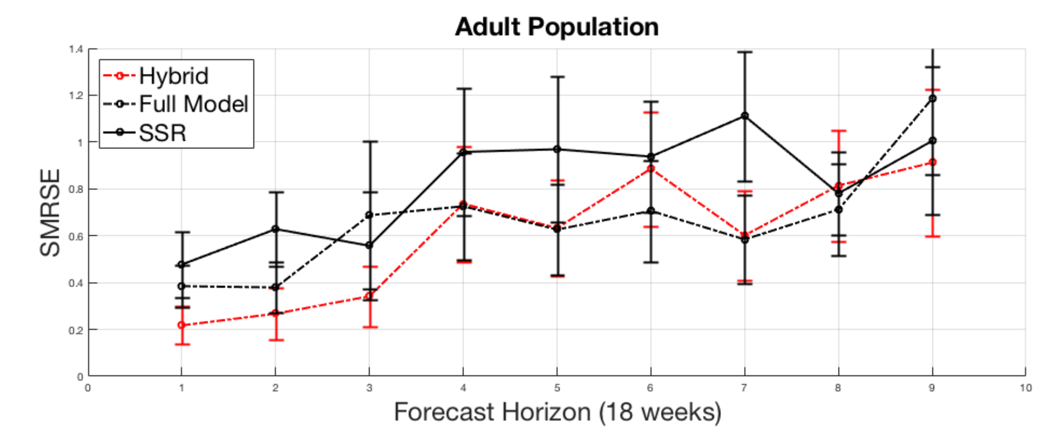


Figure: Forecast Horizon Plot for the Adult Population. The Standardized Mean Root Square Error is computed over 21 data sets using 9 time points at 2 week intervals.

- The hybrid model outperforms the full model and SSR predictions for the first several weeks.
- The uncertainty quantification for the hybrid model results in higher confidence in the forecast predictions than using the full model or SSR.

References and Acknowledgements

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