HYBRID MODELING OF CHAOTIC SYSTEMS WITH UNCERTAINTY QUANTIFICATION
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Abstract
Traditionally, modeling (parametric) and model-free (non-parametric) techniques are used for prediction, however it is uncommon for the two to be incorporated together. We compare the forecast accuracy of a Bayesian parametric methodology (DRAM) and a non-parametric approach (SSR) against each other as well as against a hybrid composed of the the two on a chaotic coupled dynamical system. We apply our hybrid approach on an age-structured population system using data from cannibalistic flour beetles, in which it has been observed that the adults preying on the eggs and pupae results in chaotic population dynamics.

Non-parametric: State Space Reconstruction
- Takens’ Theorem gives a one-to-one mapping between the attractor manifold M of the full system, and its reconstructed “shadow” manifolds. [1]
- M is one-to-one with its shadow manifolds. Hence, the shadow manifolds can be used for forecasting the future state.
- We use a nearest neighbor approach to estimate future system states directly from data without using a mechanistic model.

Parametric: Bayesian Estimation with DRAM
- The Metropolis-Hastings type Markov chain Monte Carlo (MCMC) algorithms employ a Bayesian methodology and use delayed rejection and adaptive metropolis (DRAM) to obtain posterior distributions for the modeled variable parameters. [2]

Cannibalistic Beetle Population
Age-structured Population Model
\[ L(t) = A(t-1) \exp(-C_{pa}L(t-1) - C_{ea}A(t-1)) \]
\[ P(t) = L(t-1)(1 - \mu_A) \]
\[ A(t) = P(t-1) \exp(-C_{pa}A(t-1)) + A(t-1)(1 - \mu_A) \]

Experimental Data
- Experimentally altered adult mortality, \( C_{ea} \), to 7 different values, resulting in chaotic data sets. [3]
- 3 replicates per \( C_{ea} \), total of 21 data sets.
- 41 time points per data set, sampled every 2 weeks.
- Total counts of Larvae, Pupae, and Adults.

Hybrid Methodology
Method Outline
- The hybrid prediction for A uses a partial model for A and data or SSR for L and P, where
\[ A(t) = P(t-1) \exp(-C_{pa}A(t-1)) + A(t-1)(1 - \mu_A) \]

UQ for Hybrid Prediction
- Jointly sample from the posterior distributions of the parameters for the modeled variables and the SSR sample space.

Full Model Results
- Prediction for an unmodeled state \( x_n(t) \) is made by computing a weighted average of the target variable over its nearest neighbors:
\[ \hat{x}_n(t+h) = \frac{\sum_{i=1}^{N} w_i(t) x_{n(i)}(t+h)}{\sum_{i=1}^{N} w_i(t)} \]
where \( \text{var}(x_n(t+h)) = E[(x_n(t+h) - \hat{x}_n(t))^2] \).

Jointly sample from the posterior distributions of the parameters for the modeled variables and the SSR sample space.

Forecast Comparison
- The hybrid model outperforms the full model and SSR predictions for the first several weeks.
- The uncertainty quantification for the hybrid model results in higher confidence in the forecast predictions than using the full model or SSR.

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